The θ-twistor versus the supertwistor

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Abstract

We introduce the θ-twistor which is a new supersymmetric generalization of the Penrose twistor and is also alternative to the supertwistor. The θ-twistor is a triple of *spinors* including the spinor θ extending the Penrose's double of spinors. Using the θ-twistors yields an infinite chain of massless higher spin chiral supermultiplets $(\frac{1}{2}, 1)$, $(1, \frac{3}{2})$, $(\frac{3}{2}, 2)$, ..., $(S, S + \frac{1}{2})$ generalizing the known scalar $(0, \frac{1}{2})$ supermultiplet.

1 Introduction

The (super)twistor tools [1], [2], [3] essentialy simplify the calculation of multigluon amplitudes [4], [5], [6], [7] and sharpen a traditional interest to the search of the twistor role in physics and mathematics [8], [9], [10], [11], [12], [13], [14]. The supertwistor [2] is a triple including two commuting spinors and the anticommuting scalar $\eta = \nu^\alpha \theta_\alpha$ fixing the contribution of the spinor superspace coordinate θ_α by only its projection on the Penrose spinor ν^α [1]. As a result, the chiral supermultiplet in the supertwistor description looses its auxiliary F-field and supersymmetry transformations close on the mass-shell of the Dirac field. This problem is solved here using an alternative way for the twistor supersymmetrization that preserves all the θ_α components by the introducing a new super triple in $D = 4\, N = 1$ superspace named the θ-twistor who includes *three* spinors forming a nonlinear supersymmetry representation [15]. We establish that both the θ-twistor and supertwistor appear as the general solutions of two different supersymmetric and Lorentz covariant constraints generalizing the known chirality constraint in the superspace extended by the Penrose's spinor ν_α. Using the θ-twistor restores the desired F-field at the chiral supermultiplet $(0, \frac{1}{2})$. Moreover, it makes possible to reveal an infinite chain of massless higher spin chiral supermultiplets $(\frac{1}{2}, 1)$, $(1, \frac{3}{2})$, $(\frac{3}{2}, 2)$, ..., $(S, S + \frac{1}{2})$ generalizing the well-known scalar supermultiplet $(0, \frac{1}{2})$.

2 The supertwistor

A commuting Weyl spinor ν_α belonging to the Penrose's spinor double $(\nu_\alpha, \bar{\omega}^{\dot{\alpha}})$ is inert under the transformations of $D = 4\, N = 1$ supersymmetry

$$\delta\theta_\alpha = \varepsilon_\alpha, \quad \delta x_{\alpha\dot{\alpha}} = 2i(\varepsilon_\alpha \bar{\theta}_{\dot{\alpha}} - \theta_\alpha \bar{\varepsilon}_{\dot{\alpha}}), \quad \delta\nu_\alpha = 0. \tag{1}$$

The supertwistor [2] is naturally introduced starting from the complex superspace $(y_{\alpha\dot{\alpha}}, \theta_\alpha, \bar{\theta}_{\dot{\alpha}})$ and using the supersymmetric Cartan-Volkov differential form $\omega_{\alpha\dot{\alpha}}$

$$\omega_{\alpha\dot{\alpha}} = dy_{\alpha\dot{\alpha}} + 4i d\theta_\alpha \bar{\theta}_{\dot{\alpha}}, \quad y_{\alpha\dot{\alpha}} \equiv x_{\alpha\dot{\alpha}} - 2i\theta_\alpha \bar{\theta}_{\dot{\alpha}}. \tag{2}$$

If we have the invariant vector differential one-form $\omega_{\alpha\dot\alpha}$ (2) and Weyl spinors $\nu_\alpha, \bar\nu_{\dot\alpha}$ one can construct the scalar invariant form $s = (\nu\omega\bar\nu)$ [16],[17] that may be presented as

$$s \equiv (\nu\omega\bar\nu) = s(Z, d\bar Z) = -iZ_{\mathcal{A}} d\bar Z^{\mathcal{A}}, \tag{3}$$

where the triples $Z_{\mathcal{A}}$ and $\bar Z^{\mathcal{A}}$ unify $\nu^\alpha, \bar\nu_{\dot\alpha}$ with the composite coordinates $q_\alpha, \bar q_{\dot\alpha}, \eta, \bar\eta$

$$\begin{gathered} Z_{\mathcal{A}} \equiv (-iq_\alpha, \bar\nu^{\dot\alpha}, 2\bar\eta), \quad \bar Z^{\mathcal{A}} \equiv (\nu^\alpha, i\bar q_{\dot\alpha}, 2\eta), \\ \eta \equiv \nu^\alpha\theta_\alpha, \quad \bar q_{\dot\alpha} = (q_\alpha)^* \equiv \nu^\alpha y_{\alpha\dot\alpha} = \nu^\alpha x_{\alpha\dot\alpha} - 2i\eta\bar\theta_{\dot\alpha}. \end{gathered} \tag{4}$$

The triples $Z_{\mathcal{A}}$ and $\bar Z^{\mathcal{A}}$ coincide with the supertwistor and its c.c. first proposed in [2] to be a supersymmetric generalization of the projective Penrose twistor. Then the supertwistor space is defined as a complex projective superspace equipped with the invariant bilinear form $s(Z, \bar Z')$

$$s(Z, \bar Z') \equiv -iZ_{\mathcal{A}}\bar Z'^{\mathcal{A}} = -q_\alpha\nu'^\alpha + \bar\nu^{\dot\alpha}\bar q'_{\dot\alpha} - 4i\bar\eta\eta' = 0, \tag{5}$$

where the triple $\bar Z'^{\mathcal{A}}$ is given by (4) with ν' substituted for ν

$$\bar Z'^{\mathcal{A}} \equiv (\nu'^\alpha, i\bar q'_{\dot\alpha}, 2\eta'), \quad \bar q'_{\dot\alpha} = \nu'^\alpha y_{\alpha\dot\alpha}, \quad \eta' = \nu'^\alpha\theta_\alpha. \tag{6}$$

It was shown in [2] that the null form (5) is invariant under the superconformal symmetry. Below we propose an alternative supersymmetric generalization of the Penrose twistor.

3 The θ-twistor

Here we introduce an alternative supersymmetric triple including *three* spinors. This possibility is provided by the existence of the new composite spinor l_α produced by the right multiplication of $y_{\alpha\dot\alpha}$ (2) by $\bar\nu^{\dot\alpha}$ contrarily to the left multiplication generating $\bar q_{\dot\alpha}$ (4)

$$l_\alpha \equiv y_{\alpha\dot\alpha}\bar\nu^{\dot\alpha} = x_{\alpha\dot\alpha}\bar\nu^{\dot\alpha} - 2i\theta_\alpha\bar\eta, \quad l_\alpha = q_\alpha - 4i\theta_\alpha\bar\eta. \tag{7}$$

The transformation law of l_α (7) under the supersymmetry (1) is nonlinear

$$\delta l_\alpha = -4i\theta_\alpha(\bar\nu^{\dot\beta}\bar\varepsilon_{\dot\beta}), \quad \delta\theta_\alpha = \varepsilon_\alpha, \quad \delta\bar\nu_{\dot\alpha} = 0 \tag{8}$$

and yields a new supersymmetry representation formed by a complex spinor triple $\Xi_{\mathcal{A}}$

$$\Xi_{\mathcal{A}} \equiv (-il_\alpha, \bar\nu^{\dot\alpha}, \theta^\alpha), \quad \bar\Xi^{\mathcal{A}} \equiv (\Xi_{\mathcal{A}})^* = (\nu^\alpha, i\bar l_{\dot\alpha}, \bar\theta^{\dot\alpha}). \tag{9}$$

which we call the θ-twistor. The square form (5) expressed in terms of $\Xi_{\mathcal{A}}$ and $\bar\Xi^{\mathcal{A}}$ (9)

$$\begin{gathered} s(Z, \bar Z) \equiv -iZ_{\mathcal{A}}\bar Z'^{\mathcal{A}} = \tilde s(\Xi, \bar\Xi'), \\ \tilde s(\Xi, \bar\Xi') \equiv -i\Xi_{\mathcal{A}} \circ \bar\Xi'^{\mathcal{A}} = -l_\alpha\nu'^\alpha + \bar\nu^{\dot\alpha}\bar l'_{\dot\alpha} - ig_{\alpha\dot\alpha}\theta^\alpha\bar\theta^{\dot\alpha} = 0, \quad g_{\alpha\dot\alpha} \equiv 4\nu'_\alpha\bar\nu_{\dot\alpha} \end{gathered} \tag{10}$$

becomes a nonlinear form in the θ-twistor space invariant under supersymmetry and Lorentz symmetry. The generators of the supersymmetry transformations (8) take the form

$$Q^\alpha = \tfrac{\partial}{\partial\theta_\alpha} + 4i\nu^\alpha(\bar\theta_{\dot\beta}\tfrac{\partial}{\partial\bar l_{\dot\beta}}), \quad \bar Q^{\dot\alpha} \equiv -(Q^\alpha)^* = \tfrac{\partial}{\partial\bar\theta_{\dot\alpha}} + 4i\bar\nu^{\dot\alpha}(\theta_\beta\tfrac{\partial}{\partial l_\beta}) \tag{11}$$

with their anticommutator closed by the vector generator $P^{\dot\beta\alpha} = (\bar\nu^{\dot\beta}\frac{\partial}{\partial l_\alpha} + \nu^\alpha\frac{\partial}{\partial \bar l_{\dot\beta}})$

$$\{Q^\alpha, \bar Q^{\dot\beta}\} = 4iP^{\dot\beta\alpha}, \quad [Q^\gamma, P^{\dot\beta\alpha}] = [\bar Q^{\dot\gamma}, P^{\dot\beta\alpha}] = \{Q^\gamma, Q^\beta\} = \{\bar Q^{\dot\gamma}, \bar Q^{\dot\beta}\} = 0. \tag{12}$$

It is easy to see that the nonlinear null form (10) defining the Ξ-space is also invariant under the scaling and phase transformations of the θ-twistor components

$$\begin{gathered} l'_\beta = e^{\varphi} l_\beta, \quad \bar l'_{\dot\beta} = e^{\varphi^*}\bar l_{\dot\beta}, \quad \nu'_\beta = e^{-\varphi}\nu_\beta, \quad \bar\nu'_{\dot\beta} = e^{-\varphi^*}\bar\nu_{\dot\beta}, \\ \theta'_\beta = e^{\varphi}\theta_\beta, \quad \bar\theta'_{\dot\beta} = e^{\varphi^*}\bar\theta_{\dot\beta}, \end{gathered} \tag{13}$$

described by the complex parameter $\varphi = \varphi_R + i\varphi_I$. Also (10) is invariant under the independent γ_5 rotations of θ_β and $\bar\theta_{\dot\beta}$

$$\theta'_\beta = e^{i\lambda}\theta_\beta, \quad \bar\theta'_{\dot\beta} = e^{-i\lambda}\bar\theta_{\dot\beta}. \tag{14}$$

There is an alternative way to reveal the supertwistors and the θ-twistors. One can observe that supertwistor appears as the general solution of the supersymmetric constraints

$$\begin{gathered} \bar D^{\dot\alpha} F(x,\theta,\bar\theta) = 0 \longrightarrow F = F(y,\theta), \\ \nu_\alpha D^\alpha F(y,\theta,\nu) = 0 \longrightarrow F = F(\bar Z^{\mathcal A}) \end{gathered} \tag{15}$$

in the chiral superspace ($y_{\alpha\dot\alpha}$, θ_α) complemented by the even spinor ν_α, where $F(\bar Z^{\mathcal A})$ is superfield depending on the supertwistor $\bar Z^{\mathcal A}$. Conversely, the θ-twistor is associated with the general solution of other supersymmetric constraints in the chiral space complemented by $\bar\nu_{\dot\alpha}$

$$\begin{gathered} \bar D^{\dot\alpha} F(x,\theta,\bar\theta) = 0 \longrightarrow F = F(y,\theta), \\ \bar\nu_{\dot\alpha}\frac{\partial}{\partial x_{\alpha\dot\alpha}} F(y,\theta,\bar\nu) = 0 \longrightarrow F = F(\Xi_{\mathcal A}). \end{gathered} \tag{16}$$

The object of our further investigation is the space of superfunctions $F(\Xi_{\mathcal A})$ (16) depending on the Ξ-triple.

4 Massless chiral supermultiplets of higher spin fields

The superfields $F(\bar Z^{\mathcal A})$ and $F(\Xi_{\mathcal A})$ describe massless supermultiplets because they satisfy to the Klein-Gordon equations

$$\partial_m\partial^m F(\bar Z) = 0, \quad \partial_m\partial^m F(\Xi) = 0, \tag{17}$$

where $\partial_m \equiv (\sigma_m)_{\dot\alpha\alpha}\partial^{\dot\alpha\alpha} \equiv (\sigma_m)_{\alpha\dot\alpha}\frac{\partial}{\partial x_{\alpha\dot\alpha}}, \quad \partial^{\dot\alpha\alpha} = -\frac{1}{2}\tilde\sigma_m^{\dot\alpha\alpha}\partial^m$. The component expansion of the analytical function $F(\Xi)$ in the Ξ-triple space defined by

$$F(\Xi) \equiv F(-il_\alpha, \bar\nu^{\dot\alpha}, \theta^\alpha) = f_0(-iy_{\beta\dot\beta}\bar\nu^{\dot\beta}, \bar\nu^{\dot\beta}) - 2\theta_\lambda f^\lambda(-iy_{\beta\dot\beta}\bar\nu^{\dot\beta}, \bar\nu^{\dot\beta}) + \theta^2 f_2(-iy_{\beta\dot\beta}\bar\nu^{\dot\beta}, \bar\nu^{\dot\beta}), \tag{18}$$

where $\theta_\alpha\theta_\beta = \frac{1}{2}\varepsilon_{\alpha\beta}\theta^2$, preserves the desired auxiliary field f_2 vanishing in the supertwistor description. The contour integral generalizing the Penrose's supertwistor integral is given by

$$\Phi^{\dot\alpha_1\ldots\dot\alpha_{2S}}(y,\theta) = \oint (d\bar\nu^{\dot\gamma}\bar\nu_{\dot\gamma})\bar\nu^{\dot\alpha_1}\ldots\bar\nu^{\dot\alpha_{2S}} F(\bar\nu^{\dot\beta}, -i\bar\nu^{\dot\gamma}y_{\beta\dot\gamma}, \theta_\beta), \tag{19}$$

where $F(\Xi)$ is supposed to have the degree of homogeneity equal $-2(S+1)$ and the $\bar\nu$-contour encloses the singularities of F for each fixed point (y,θ). Inserting (18) into (19) we get

$$\Phi^{\dot\alpha_1...\dot\alpha_{2S}}(y,\theta) = f_0^{\dot\alpha_1...\dot\alpha_{2S}}(y) - 2\theta_\lambda f^{\lambda\dot\alpha_1...\dot\alpha_{2S}}(y) + \theta^2 f_2^{\dot\alpha_1...\dot\alpha_{2S}}(y) \tag{20}$$

where

$$\begin{aligned} f_0^{\dot\alpha_1...\dot\alpha_{2S}}(y) &= \oint (d\bar\nu^{\dot\gamma}\bar\nu_{\dot\gamma})\bar\nu^{\dot\alpha_1}...\bar\nu^{\dot\alpha_{2S}} f_0(-iy_{\beta\dot\beta}\bar\nu^{\dot\beta},\bar\nu^{\dot\beta}),\\ f^{\lambda\dot\alpha_1...\dot\alpha_{2S}}(y) &= \oint (d\bar\nu^{\dot\gamma}\bar\nu_{\dot\gamma})\bar\nu^{\dot\alpha_1}...\bar\nu^{\dot\alpha_{2S}} f^\lambda(-iy_{\beta\dot\beta}\bar\nu^{\dot\beta},\bar\nu^{\dot\beta}),\\ f_2^{\dot\alpha_1...\dot\alpha_{2S}}(y) &= \oint (d\bar\nu^{\dot\gamma}\bar\nu_{\dot\gamma})\bar\nu^{\dot\alpha_1}...\bar\nu^{\dot\alpha_{2S}} f_2(-iy_{\beta\dot\beta}\bar\nu^{\dot\beta},\bar\nu^{\dot\beta}) \end{aligned} \tag{21}$$

with $f^{\lambda\dot\alpha_1...\dot\alpha_{2S}}(y)$ and $f_{0,2}^{\dot\alpha_1...\dot\alpha_{2S}}(y)$ satisfying the chiral Dirac equations

$$\partial_{\alpha\dot\alpha_k} f^{\lambda\dot\alpha_1..\dot\alpha_k..\dot\alpha_{2S}}(x) = \partial_{\alpha\dot\alpha_k} f_{0,2}^{\dot\alpha_1..\dot\alpha_k..\dot\alpha_{2S}}(x) = 0, \quad (k=1,2,...,2s). \tag{22}$$

The further expansion of $\Phi^{\dot\alpha_1...\dot\alpha_{2S}}(y,\theta)$ (20) at the point x_m is given by

$$\begin{aligned} \Phi^{\dot\alpha_1...\dot\alpha_{2S}}(y,\theta) = f_0^{\dot\alpha_1...\dot\alpha_{2S}}(x) - 2\theta_\lambda f^{\lambda\dot\alpha_1...\dot\alpha_{2S}}(x) - 2i\theta_\gamma\bar\theta_{\dot\gamma}\partial^{\dot\gamma\gamma} f_0^{\dot\alpha_1...\dot\alpha_{2S}}(x)\\ -2i\theta^2\bar\theta_{\dot\gamma}\partial^{\dot\gamma\lambda} f_\lambda^{\dot\alpha_1...\dot\alpha_{2S}}(x) + \theta^2 f_2^{\dot\alpha_1...\dot\alpha_{2S}}(x), \end{aligned} \tag{23}$$

where the term $\frac{1}{2}\theta^2\bar\theta^2\partial^{\dot\gamma\gamma}\partial_{\gamma\dot\gamma} f_0^{\dot\alpha_1...\dot\alpha_{2S}}(x)$ was droped because of the zero mass constraint (17)

$$\Box\Phi^{\dot\alpha_1...\dot\alpha_{2S}}(y,\theta) = 0 \longrightarrow \Box f_0^{\dot\alpha_1...\dot\alpha_{2S}}(x) = 0. \tag{24}$$

For sewing together of these results with the well-known case $S=0$ corresponding to the scalar supermultiplet let us rename the component f-fields by the letters used in [18]

$$\begin{aligned} f_0^{\dot\alpha_1...\dot\alpha_{2S}} = \sqrt{2}A^{\dot\alpha_1...\dot\alpha_{2S}} \equiv \sqrt{2}A^{\cdots}, \quad f_2^{\dot\alpha_1...\dot\alpha_{2S}} = \sqrt{2}F^{\dot\alpha_1...\dot\alpha_{2S}} \equiv \sqrt{2}F^{\cdots},\\ f_\lambda^{\dot\alpha_1...\dot\alpha_{2S}} = \psi_\lambda^{\dot\alpha_1...\dot\alpha_{2S}} \equiv \psi_\lambda^{\cdots}, \end{aligned} \tag{25}$$

where $(...) \equiv (\dot\alpha_1...\dot\alpha_{2S})$. Then we find the superfield $\frac{1}{\sqrt2}\Phi^{\cdots}(y,\theta)$ to describe the massless chiral multiplet [18] for the case $S=0$. For $S\neq 0$ the superfield (20) represents the chiral supermultiplets of massless higher spin fields with the particle spin content

$$(\frac{1}{2},1),\ (1,\frac{3}{2}),\ (\frac{3}{2},2),\ ....,(S,S+\frac{1}{2})$$

accompanied by the corresponding auxiliary fields for any integer or half-integer spin $S=\frac{1}{2},1,\frac{3}{2},2,.....$ The supersymmetry transformations for the higher spin multiplet (20) presented in the notations (25) take the form

$$\begin{aligned} \delta A^{\cdots} = \sqrt{2}\varepsilon^\lambda\psi_\lambda^{\cdots}, \quad \delta F^{\cdots} = i\sqrt{2}(\bar\varepsilon\tilde\sigma_m\partial^m\psi^{\cdots})\\ \delta\psi_\lambda^{\cdots} = i\sqrt{2}(\sigma_m\bar\varepsilon)_\lambda\partial^m A^{\cdots} + \sqrt{2}\varepsilon_\lambda F^{\cdots} \end{aligned} \tag{26}$$

and coincide with the transformation rules for the $S=0$ chiral multiplet of the weight $n=\frac{1}{2}$ [18] if we put $A^{\cdots}=A$, $F^{\cdots}=F$ and $\psi_\lambda^{\cdots}=\psi_\lambda$ in the relations (26).

As it was noted the θ-twistor superspace is invariant under the axial rotations (14) and one can consider these phase transformations as inducing the R-symmetry transformations for the superfield $F(\Xi)$ (18)

$$F'(-il_\alpha,\bar\nu^{\dot\alpha},e^{i\varphi}\theta^\alpha) = e^{2in\varphi}F(-il_\alpha,\bar\nu^{\dot\alpha},\theta^\alpha), \tag{27}$$

where n is the correspondent R number. Then taking into account the representation (19) we get the R-symmetry transformation of the generalized chiral superfield $\Phi^{\dot{\alpha}_1\ldots\dot{\alpha}_{2S}}(y,\theta)$

$$\Phi'^{\dot{\alpha}_1\ldots\dot{\alpha}_{2S}}(y,\theta) = e^{2in\varphi}\Phi^{\dot{\alpha}_1\ldots\dot{\alpha}_{2S}}(y, e^{-i\varphi}\theta). \tag{28}$$

So, one can expect that new renormalizable Lagrangians may be constructed using these higher spin superfields.

5 Conclusion

Proposed is the new supersymmetric generalization of the Penrose twistor generating the θ-twistor. As a new mathematical object the θ-twistor deserves to be studied both in its own rights and in further physical applications. A creative charge of the θ-twistor was here illustrated by the production of an infinite chain of higher spin chiral supermultiplets generalizing the massless scalar supermultiplet. The new chiral superfields may be used for the construction of new physically interesting models because the studied here $D = 4\, N = 1$ example has the direct generalization both to the case of extended supersymmetries by the change $(\theta_\alpha, \bar{\theta}_{\dot{\alpha}}) \to \theta^i_a$ and to higher dimensions $D = 2, 3, 4(mod 8)$ by analogy with the supertwistors [2], [3], [19].

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References

[1] R. Penrose and W. Rindler, *Spinor and twistor methods in space-time geometry.* Cambridge University Press, 1986.

[2] A. Ferber, Nucl. Phys. **B 132** (1978) 55.

[3] E. Witten, Phys. Lett. **B77** (1978) 215.

[4] E. Witten, Commun. Math. Phys. **252** (2004) 189, arXiv: hep-th/0310284.

[5] V.P. Nair, *Noncommutative mechanics, Landau levels, twistors and Yang-Mills amplitudes*, arXiv: hep-th/0506120.

[6] R. Roiban, M. Spradlin and A. Volovich, JHEP **0404** 012 (2004).

[7] G. Georgiou and V.V. Khoze, JHEP **0405** 070 (2004).

[8] M.F. Atiyah, V.G. Drinfeld, N.J. Hitchin, Yu.I. Manin, Phys. Lett. **65A** (1978) 185.

[9] E. Corrigan, D. Fairlie, P. Goddard, and S. Templeton, Nucl. Phys. **B140** (1978) 31.

[10] T. Shirafuji, Prog. Theor. Phys. **70** (1983) 18.

[11] I. Bengtsson and M. Cederwal, Nucl. Phys. **B 302** (1988) 81.

[12] N. Berkovits, Phys. Rev. Lett. **93** (2004) 011601.

[13] W. Siegel, *Untwisting the twistor superstring*, arXiv: hep-th/0404255.

[14] W. Siegel, Phys. Rev. **D52** (1995) 1042.

[15] A.A. Zheltukhin, *A new type of supersymmetric twistors and higher spin chiral multiplets*, arXiv: hep-th/0606234.

[16] D.V. Uvarov and A.A. Zheltukhin, JHEP **03** (2004) 063.

[17] A.A. Zheltukhin, Mod. Phys. Lett. **A 21** (2006) 2117.

[18] J. Wess and J. Bagger, *Supersymmetry and Supergravity*. Princeton University Press, 1992.

[19] I. Bengtsson and A.A. Zheltukhin, Phys. Lett. **B 570** (2003) 222.

[20] S. Fedoruk, E. Ivanov and J. Lukierski, Phys. Lett. **B 641** (2006) 226.